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## STRUCTURAL SENSITIVITY ANALYSIS: METHODS, APPLICATIONS AND NEEDS

HOWARD M. ADELMAN, RAPHAEL T. HAFTKA,  
CHARLES J. CAMARDA, AND JOANNE L. WALSH

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National Aeronautics and  
Space Administration

Langley Research Center  
Hampton, Virginia 23665

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## INTRODUCTION

The field of sensitivity derivative analysis is emerging as one of the more fruitful areas of engineering research. The reason for this is the recognition of the many practical uses for sensitivity derivatives. Beyond the historical use of derivatives in connection with formal mathematical optimization techniques, recent work has been reported in using sensitivity derivatives in approximate analysis, assessing design trends, analytical model improvement, and determining effects of parameter uncertainties (refs. 1 through 7).

Work supported by the NASA Langley Research Center under grant in sensitivity analysis has been focused on derivatives of thermal response of structures (refs. 8,9). Most recently, in-house implementations of generalized structural sensitivity capability in the SPAR and EAL computer programs (ref. 10 and 11) have been completed. Work in the sensitivity area is being expanded and recent developments both in and outside the structures area have been surveyed to guide the future effort. This paper reviews some innovative techniques applicable to sensitivity analysis of discretized structural systems. The techniques include a finite difference step size selection algorithm, a method for derivatives of iterative solutions, a Green's function technique for derivatives of transient response, simultaneous calculation of temperatures and their derivatives, derivatives with respect to shape, and derivatives of optimum designs with respect to problem parameters. Computerized implementations of sensitivity analysis and applications of sensitivity derivatives are also discussed. Finally, some of the critical needs in the structural sensitivity area are indicated along with Langley plans for dealing with some of those needs.

### ● USES OF SENSITIVITY DERIVATIVES

- FORMAL OPTIMIZATION ALGORITHMS
- APPROXIMATE ANALYSIS
- DESIGN TRENDS
- ANALYTICAL MODEL IMPROVEMENT
- EFFECTS OF PARAMETER UNCERTAINTIES

### ● CONTRIBUTIONS FROM BROAD GROUP OF DISCIPLINES

### ● REVIEW OF SELECTED TECHNIQUES

### ● APPLICATIONS

### ● NEEDS AND PLANS

## DISCIPLINES CONTRIBUTING TO SENSITIVITY ANALYSIS DEVELOPMENT

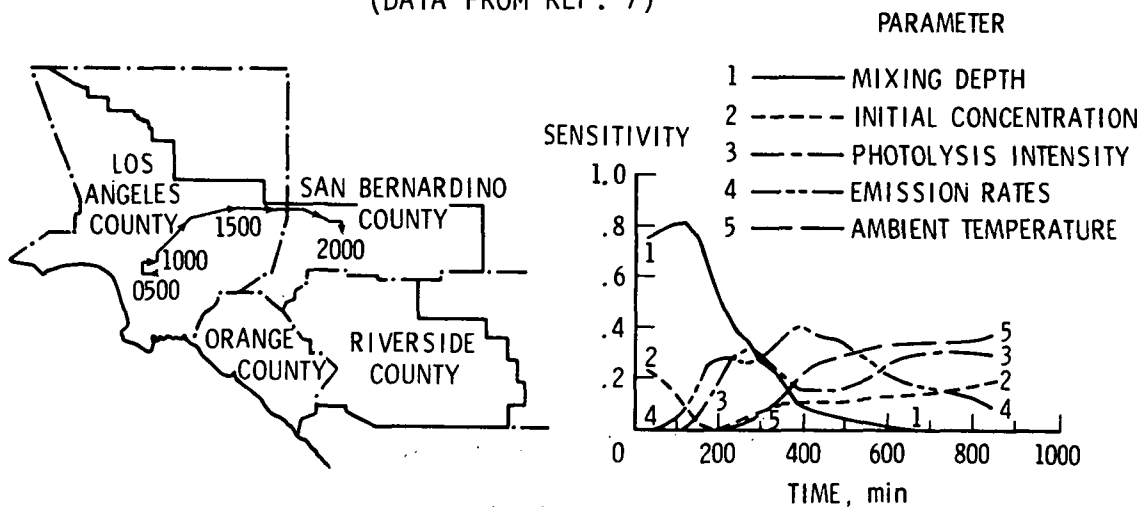
Sensitivity methodology has been and continues to be an important research area for many disciplines. Appreciation for the uses of sensitivity analysis by a broad spectrum of researchers outside the structures area is very evident. Some of those disciplines are indicated in the figure. For the most part, the motivation in these other disciplines is the need to quantify the effect of uncertainties in parameters of a system model on the predictions of the model. Examples from physical chemistry are described in refs. 7, 12-18. A specific use will be given in the next figure. Electronics and control theory represented the origin of this type of sensitivity work (refs. 19, 20) in addition to the use of derivatives to synthesize systems. Recent work in physiology with both human and bacteriological system models is described in reference 21. In the thermodynamics area, reference 22 describes the calculations and use of derivatives of the chemical composition with respect to thermodynamic properties in the mathematical modeling of a coal gasification process. Finally analytical techniques are beginning to emerge to calculate derivatives of aerodynamic quantities with respect to flow parameters (refs. 23 and 24) as described in the next paper in this document (ref. 25). This paper focuses on contributions to sensitivity methodology originating in or applicable to the structural analysis field.

- CHEMICAL KINETICS
- ELECTRONICS AND CONTROL
- PHYSIOLOGY
- THERMODYNAMICS
- AERODYNAMICS
- STRUCTURAL ANALYSIS

## APPLICATION OF SENSITIVITY DERIVATIVES TO ATMOSPHERIC POLLUTION MODEL

Sensitivity analysis has been used to assess the effects of uncertainties in emission and meteorological parameters on the predictions from a mathematical model for photochemical air pollution (ref. 7). The atmospheric diffusion equation which governs the degree of pollution of a volume of air (an air parcel) contains several parameters: mixing depth - the vertical height of the air parcel containing pollutants; initial concentration of pollutant; photolysis intensity - rate of photochemical activity; emission rate - the rate at which the pollutant is emitted into the air parcel; and ambient temperature of the air. The calculation of derivatives of concentrations of various pollutants with respect to the aforementioned parameters is described in reference 7. The derivatives were used to rank the importance of the parameters. The calculations were carried out for the example of an "oxidant episode" which occurred in Southern California in 1974. The mathematical simulation of the event began in downtown Los Angeles at 5 a.m. and terminated in San Bernardino County at 8 p.m. The graph shows the sensitivity of the concentration of the pollutant Nitric Oxide (NO) with respect to each parameter as a function of time. Results indicate that early in the episode the initial concentration and mixing depth are the most important parameters. Midway through, emission rate and ambient temperature were most important and late in the calculation, ambient temperature and photolysis intensity were most critical. These types of data indicated the need for more exact measurements of the key parameters to improve the air quality mathematical model.

(DATA FROM REF. 7)



EXAMPLE PROBLEM —  
POLLUTION DUE TO OXIDANT EPISODE

SENSITIVITY OF NO CONCENTRATION

- STUDY RESULTS LED TO RESEARCH FOR MORE  
PRECISE MEASUREMENTS OF KEY PARAMETERS

## OPTIMUM STEP SIZE FOR FINITE-DIFFERENCE DERIVATIVES

The most straight forward method of calculating derivatives is to use a finite-difference approximation. One of the most serious shortcomings of the finite-difference method is the uncertainty in the choice of a perturbation step size. If the step size is too large, truncation errors may occur. These can be thought of as errors due to retention of only the lowest order terms of a Taylor series representation of a perturbed function. If the step size is too small, condition errors may occur (ref. 26). These errors are due to subtraction of nearly equal numbers. In a recent paper (ref. 27) an algorithm was developed to determine the optimum finite-difference step size i.e. one which balances the truncation and condition errors. The algorithm is based on approximating the truncation error as a linear function of step size  $h$  and the condition error as a linear function of  $1/h$ . The optimum step size is obtained by equating the condition and truncation errors. This technique has been tested on functions which could be differentiated analytically (ref. 27) and was found to be very effective. A logical extension of this work would be to apply it to matrix equations.

$$\bullet \text{ WANT BEST ESTIMATE OF } \frac{\partial f}{\partial v} \approx \frac{1}{h} (f(v+h) - f(v))$$

THE PROBLEM  $\left\{ \begin{array}{l} \text{IF } h \text{ TOO LARGE} - \text{TRUNCATION ERROR} \equiv T(h) \\ \text{IF } h \text{ TOO SMALL} - \text{CONDITION ERROR} \equiv C(h) \end{array} \right.$

THE SOLUTION  $\left\{ \begin{array}{l} \text{EXPRESS } T(h) \text{ AND } C(h) \text{ AS SIMPLE COMPUTABLE FUNCTIONS} \\ \text{CHOOSE "OPTIMUM" STEP SIZE } \hat{h} \text{ SO THAT} \end{array} \right.$

$$C(\hat{h}) = T(\hat{h})$$

- RESULT — FORMULA FOR  $\hat{h}$
- FORMULA VERIFIED BY TESTS ON ANALYTICAL FUNCTIONS
- NEED TO IMPLEMENT FOR MATRIX EQUATIONS

## DERIVATIVES OF ITERATIVE SOLUTIONS

In many structural design problems the response  $U$  is the solution of an algebraic system  $f(U, v) = 0$  where  $v$  is a design parameter. When the system is solved iteratively, the iterative process is terminated when the solution error is reduced below a certain tolerance. To obtain the derivative of  $U$  with respect to a design parameter by finite differences, the parameter is perturbed and the solution process is repeated to obtain  $U_h$ . The derivative is then approximated by a finite difference ratio. The error inherent in the above process is due to the termination of the iterative solution process before an exact solution is obtained. Thus  $\bar{U}$  and  $\bar{U}_h$  obtained by iteration, are only approximations to the corresponding exact solutions,  $U$  and  $U_h$ , respectively.

Because of noise in the solution process the difference between  $\bar{U}$  and  $\bar{U}_h$  can be finite even for very small values of the perturbation,  $h$ . In fact, the error is most severe when small values of  $h$  are required to avoid large truncation errors in the derivative. A remedy to this difficulty being developed by the second author of this paper is to define a modified perturbed solution  $U_h^*$  which satisfies a modified equation whose right hand side is not zero but is the residual of the approximate unperturbed equation. By this construction  $U_h^*$  approaches  $\bar{U}$  as  $h$  approaches zero. Then  $U_h^*$  replaces  $\bar{U}_h$  in the derivative formula. Finally,  $\bar{U}$  serves as the first approximation in the iteration process for  $U_h^*$ .

- |  |   |  |
|--|---|--|
| <ul style="list-style-type: none"> <li>• <math>\bar{U}</math> IS SOLUTION TO <math>f(U, v) = 0</math></li> <li>• <math>\bar{U}_h</math> IS SOLUTION TO <math>f(U_h, v + h) = 0</math></li> </ul> | } | <p>APPROXIMATE<br/><math>\bar{U}</math> AND <math>\bar{U}_h</math><br/>OBTAINED BY ITERATION</p> |
|--|---|--|

$$\partial U / \partial v = 1/h (\bar{U}_h - \bar{U})$$

- ERROR MAY BE LARGE DESPITE SMALL  $h$
- SOLUTION — DEFINE  $U_h^*$  SUCH THAT

$$f(U_h^*, v + h) = \underbrace{f(\bar{U}, v)}_{\text{RESIDUAL}} \quad \text{THEN } \partial U / \partial v = 1/h (U_h^* - \bar{U})$$

- WHY IT WORKS      INITIAL GUESS FOR  $U_h^*$  IS  $\bar{U}$   
 $U_h^*$  APPROACHES  $\bar{U}$  FOR SMALL  $h$

## GREEN'S FUNCTION METHOD FOR DERIVATIVES OF TRANSIENT RESPONSE

This method which is well known in applications to solutions of nonhomogeneous differential equations has been used extensively by physical chemistry researchers (refs. 12 through 18) for calculation of derivatives of response quantities governed by systems of first order nonlinear ordinary differential equations such as equation (1). Numerous applications have been performed for chemical kinetics problems related to air pollution studies. As indicated in the figure, the derivative of the response vector  $Y$  with respect to a parameter  $\alpha$  satisfies equation (2). The derivative may be represented by an integral expression (eq. 3) involving a kernel  $K$  which is the Green's function. The Green's function is the solution to the initial value problem given by equation (4). Comparison of the effort needed to solve equation (2) vs. (4) indicates that the Green's function technique is advantageous if the number of design variables ( $m$ ) exceeds the number of equations in the system ( $n$ ). One approach to obtaining  $K$  is to solve equation (4) directly using an implicit numerical integration technique (ref. 12). An alternate solution for the Green's function is to use the Magnus method (ref. 17) whereby  $K$  is expressed as an exponential function of a matrix which is the time integral of the Jacobian matrix,  $J$ . Because the equation of transient heat transfer is a special case of equation (1), the Green's function method is directly applicable to sensitivity of transient temperatures. It is planned to pursue this line of research at Langley as part of our sensitivity development.

### ● IMPLEMENTED BY PHYSICAL CHEMISTRY RESEARCHERS

$$\begin{aligned} \bullet \text{ GENERAL PROBLEM } & \begin{cases} \frac{dY}{dt} = f(Y, \alpha, t) & n \text{ EQUATIONS} & (1) \\ \frac{dY_\alpha}{dt} = JY_\alpha + f_\alpha & mn \text{ EQUATIONS} & (2) \\ J \equiv \partial f / \partial Y & Y_\alpha \equiv \partial Y / \partial \alpha \end{cases} \end{aligned}$$

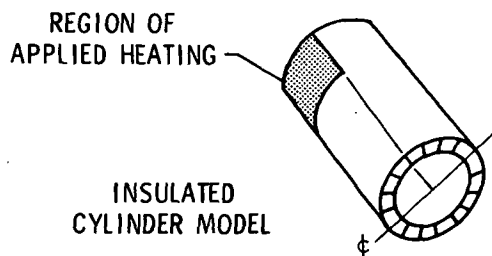
$$\begin{aligned} \bullet \text{ SOLUTION BY GREEN'S FUNCTION } & \begin{cases} Y_\alpha = \int_0^t K(t, \tau) f_\alpha d\tau & (3) \\ \frac{dK}{dt}(t, \tau) - JK(t, \tau) = 0 & n^2 \text{ EQUATIONS} & (4) \end{cases} \end{aligned}$$

### ● APPLICABLE TO TRANSIENT HEAT TRANSFER

## CONCURRENT CALCULATIONS OF TRANSIENT TEMPERATURES AND DERIVATIVES

Derivatives of transient response such as structural displacements and temperatures have been computed following the calculation of the response itself using analytical techniques (refs. 9, 28, 29) and by the Green's function technique (ref. 12). Recently an algorithm for concurrent calculation of transient temperatures and their finite difference derivatives has been developed (ref. 9). The figure depicts an application to the transient thermal response of an insulated cylindrical shell. Temperatures throughout the shell are computed using an implicit numerical integration technique. Along with temperatures at each time step, finite difference sensitivity derivatives are calculated with respect to design variables representing insulation thicknesses at 10 locations on the shell surface. The key to the success of the method is that during each time step, when a nonlinear algebraic equation is solved by iteration for the current temperature and the perturbed temperature, the same time step is used for both solutions. Further, the unperturbed temperature serves as the initial guess in the iteration for the perturbed temperature. The numbers in the table are solution times in seconds. The results indicate that the timesaving from the concurrent calculation is substantial and nearly a factor-of-four advantage is obtained.

ERROR TOLERANCE ON TEMPERATURE	SEQUENTIAL		CONCURRENT	
	CPU TIME FOR TEMPERATURE CALCULATION	CPU TIME FOR EACH DERIVATIVE	CPU TIME FOR TEMPERATURE PLUS TEN DERIVATIVES	CPU TIME FOR EACH DERIVATIVE
.001	113	113	500	39
.003	78	78	296	22
.010	46	46	168	12



TYPICAL DERIVATIVE

$$\frac{\partial T}{\partial t_{ins}}$$

T = TEMPERATURE

$t_{ins}$  = INSULATION THICKNESS

## SENSITIVITY DERIVATIVES FOR SHAPE DESIGN VARIABLES

A relatively new topic in structural sensitivity analysis is calculation of derivatives with respect to shape design variables. Examples are derivatives of displacements or stresses with respect to a beam length, or a membrane area. Two approaches have been used. One approach is to differentiate the discretized equations resulting from a finite element representation. A drawback to this technique is that when shape design variables change, the finite element mesh is modified. The resulting mesh distortion changes the discretization error and leads to inaccurate derivatives. The second approach which avoids mesh distortion errors is to reverse the order of differentiation and discretization (refs. 30-32). The procedure is to differentiate the continuum equations of equilibrium and discretize the resulting integral equations. This method uses the concept of a material derivative from continuum mechanics which is composed of two parts - a derivative corresponding to a fixed shape, and a contribution from the change of the boundary. The preferred choice between the two methods is not yet clear. The second approach avoids mesh distortion by its formulation but does not permit shape differentiation of a discretized set of equations. The first method although suffering from mesh distortion errors could benefit from a built-in adaptive mesh generation capability which would reduce the mesh distortion.

### ● EXAMPLES — DERIVATIVE OF

- |                 |         |                  |
|-----------------|---------|------------------|
| ● DISPLACEMENTS | WITH    | LENGTH OF BEAM   |
|                 | RESPECT |                  |
| ● STRESSES      | TO      | AREA OF MEMBRANE |

### ● FIRST METHOD — DISCRETIZE FIRST THEN DIFFERENTIATE

- NUMERICAL ERRORS DUE TO MESH DISTORTION
- REDUCE ERRORS BY ADAPTIVE MESH GENERATION

### ● SECOND METHOD — BASED ON MATERIAL DERIVATIVE

- DIFFERENTIATE CONTINUUM EQUATIONS THEN DISCRETIZE
- AVOIDS MESH DISTORTION ERRORS

## SENSITIVITY OF OPTIMUM DESIGNS TO PROBLEM PARAMETERS

The problem addressed by this technique (refs. 33, 34) is to obtain derivatives of an objective function ( $F$ ) and design variables ( $V$ ) from an optimized solution with respect to parameters ( $P$ ) which were held constant during the optimization. The most obvious and thus far most useful application of the technique is extrapolation of an optimum design for variations of a problem parameter. For example the effect of varying the height  $H$  of the truss in the figure is assessed by using optimum sensitivity derivatives. Extrapolated values of the mass  $F$  and one of the cross sectional areas  $A_1$  based on derivatives with respect to  $H$  are compared with those obtained by reoptimization with different values of  $H$ . As shown in the lower right portion of the figure, the results agree very closely for up to a twenty percent change in  $H$ . Other applications of these types of derivatives include optimization for multiple objective functions, assessing the effects of adding or deleting constraints, and most recently using the derivatives as links between subsystems during multilevel optimization (ref. 35).

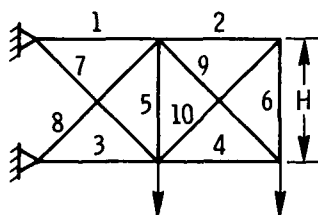
GIVEN AN OPTIMUM DESIGN:

$F$  = OBJECTIVE FUNCTION

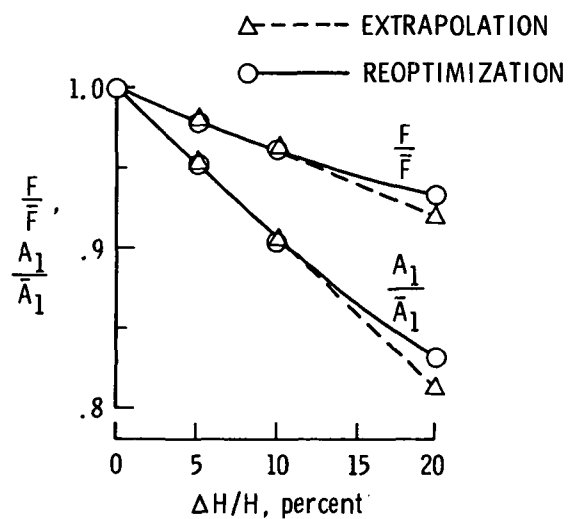
$V$  = DESIGN VARIABLES

$P$  = PROBLEM PARAMETERS

ANALYSIS GIVES  $\frac{\partial F}{\partial P}$  AND  $\frac{\partial V}{\partial P}$



● EXAMPLE — EFFECT OF TRUSS HEIGHT ON OPTIMUM DESIGN



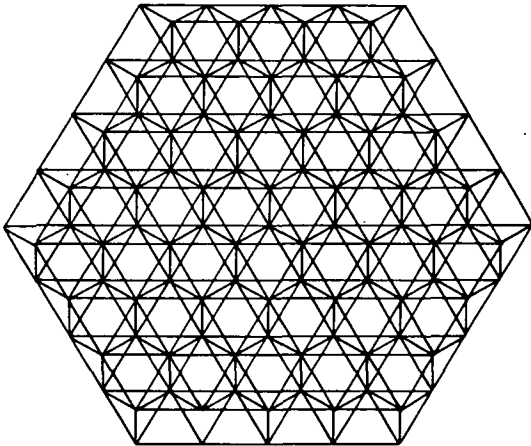
## COMPUTER IMPLEMENTATION

Some progress has occurred in providing general-purpose software for sensitivity analysis. The Green's function technique described earlier has been implemented in a computer program denoted AIM (ref. 18). Use of this program requires supplying subroutines to define the system of equations - specifically, the vector  $f$  and the matrix  $J$  in figure 5. Capability for computing derivatives of static displacements, stresses, vibration and buckling eigenvalues and eigenvectors have been implemented in the SPAR finite-element program (ref. 10) and EAL (refs. 11, 36). The EAL (Engineering Analysis Language) system contains the SPAR finite element analysis modules but additionally EAL provides FORTRAN-like commands which permit branching, testing data, looping, and calling the SPAR modules (similar to calling FORTRAN subroutines). A recent level of a proprietary version of the NASTRAN computer program also has capability for static displacement, stress and eigenvalue derivatives (ref. 37).

- AIM (GREEN'S FUNCTION TECHNIQUE)
  - GENERAL FIRST ORDER EQUATIONS
  - DERIVATIVES WRT PARAMETERS IN EQUATIONS
- SPAR (COSMIC) AND EAL (PROPRIETARY)
  - DERIVATIVES OF —
    - DISPLACEMENTS
    - STRESSES
    - EIGENVALUES
    - EIGENVECTORS
- NASTRAN (PROPRIETARY)
  - DERIVATIVES OF —
    - DISPLACEMENTS
    - STRESSES
    - EIGENVALUES

## APPLICATION OF STRUCTURAL SENSITIVITY ANALYSIS TO SPACE ANTENNA

An application of sensitivity analysis to reveal structural design trends is illustrated in this figure. The structure is an earth-orbiting antenna reflector subjected to nonuniform heating leading to thermal distortions which can degrade antenna performance (ref. 11). The structure is modeled using only rod elements. There are three design variables representing respectively, the cross-sectional areas of the elements in the upper surface ( $A_1$ ), the elements joining the upper and lower surfaces ( $A_2$ ), and the elements in the lower surface ( $A_3$ ). Derivatives of the center deflection with respect to each design variable were calculated and are shown in the figure. A positive derivative indicates that increasing the design variable increases the response. A negative derivative indicates that increasing the design variable decreases the response. The seemingly contradictory result that increasing a design variable can increase a response stems from the fact that the thermal loads are proportional to the rod cross-sectional areas. From the table we see that increasing  $A_1$  has the largest effect on reducing deflection but at the cost of a weight increase. On the other hand, decreasing either  $A_2$  or  $A_3$  would reduce the deflection and at the same time would reduce weight. It is at the discretion of the designer which of the alternatives is a better choice. The sensitivity derivatives provide the data for that judgment.



ANTENNA MODEL

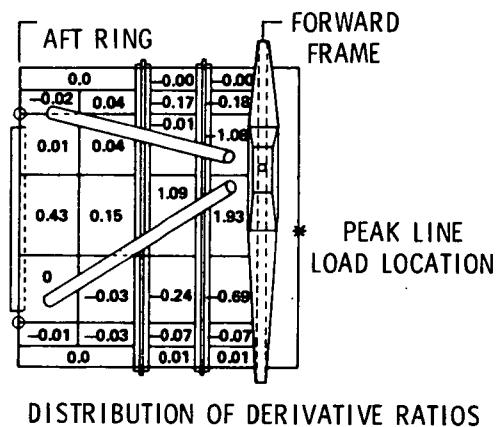
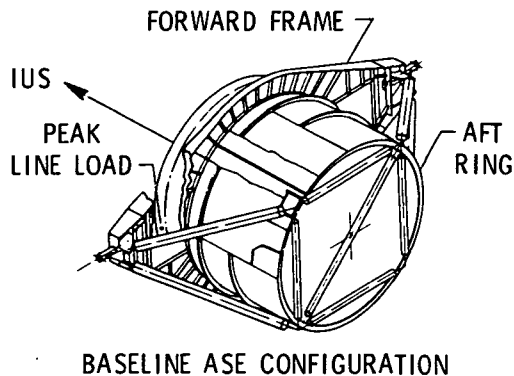
$w_0$  = CENTER DEFLECTION

DESIGN VARIABLE — ELEMENT AREA, $A_i$	$\partial w_0 / \partial A_i$
UPPER SURFACE	$-2.4 \times 10^{-4}$
DIAGONALS	$8.3 \times 10^{-5}$
LOWER SURFACE	$1.8 \times 10^{-4}$

DERIVATIVES OF CENTER DEFLECTION

## APPLICATION OF SENSITIVITY ANALYSIS TO SHUTTLE PAYLOAD

Sensitivity analysis has been used to redesign the Airborne Support Equipment (ASE) assembly on the space shuttle orbiter (ref. 5). The ASE supports the inertial upper stage (IUS) vehicle in the payload bay. The purpose of the IUS vehicle is to transport payloads further into space once the orbiter has established low earth orbit. The shell of the ASE is subjected to large launch loads and is designed for an ultimate load of 3000 lb/in. An initial sculptured skin design met the design load but was too heavy. A sensitivity analysis was performed to determine which skin gages had the largest effect on loads and to determine which type of modified construction would give the largest weight reduction (among isogrid, waffle, and stiffened skin). The structure was modeled and divided into zones as shown on the right side of the figure. Derivatives of compressive loads and weight with respect to longitudinal, circumferential and shear stiffening were computed for each zone. As an example, consider derivatives with respect to the longitudinal stiffness design variable  $t_a$ . The numbers in the zones are ratios of derivatives of load to derivatives of weight for the sculptured skin design. Negative values indicate that increasing a design variable decreases the load and positive values indicate that increasing a design variable increases the load. The analysis revealed that derivatives with respect to longitudinal and shear stiffness were the largest, derivatives with respect to circumferential stiffness were negligible, and derivatives with respect to shear stiffness were nearly all positive. Based on these results, the shell was redesigned as a longitudinally-stiffened machined skin (isogrid was rejected because of high shear stiffness; waffle construction was rejected due to unneeded high circumferential stiffness). The resulting design satisfied the ultimate load constraint with a large margin of safety and an acceptably low weight.



- STUDY RESULTS
  - LARGE DERIVATIVES WRT LONGITUDINAL AND SHEAR STIFFNESS
  - SMALL DERIVATIVES WRT CIRCUMFERENTIAL STIFFNESS
- OUTCOME — REDESIGNED SHELL AS LONGITUDINALLY-STIFFENED MACHINED PANEL SATISFIED DESIGN REQUIREMENTS WITH LOW WEIGHT

$$\frac{\partial N / \partial t_a}{\partial W / \partial t_a}$$

$t_a$  = LONGITUDINAL STIFFNESS

## STRUCTURAL SENSITIVITY ANALYSIS NEEDS

As a result of surveying methods applicable to computing structural sensitivity derivatives a list of needs has emerged. First, continued development of methods for derivatives of transient response and derivatives with respect to shape design variables and material properties should have high priority. Further, techniques developed for sensitivity derivatives in nonstructural disciplines such as physical chemistry have much to offer and should be evaluated for their adaptability to structural areas. It appears that structural designers have made insufficient use of the power and utility of sensitivity derivatives to guide design modifications and to assess uncertainties in their models. Their use can be accelerated by demonstrations of practical applications of sensitivity analysis and careful documentation (by optimization and sensitivity specialists) to guide structural analysts and designers not experienced in formal optimization and sensitivity analysis. Finally, sensitivity analysis needs to be routinely included as a standard feature in general-purpose structural analysis software packages. Near-term plans at Langley include evaluation of the Green's function method for derivatives of transient thermal response, methods for derivatives of spacecraft thermal response with respect to material properties, and implementation of the optimum finite-difference step size technique for finite-element sensitivity analysis. Concurrent with this effort, demonstration problems will be selected and solved.

- TRANSIENT RESPONSE
- MATERIAL PROPERTIES AND SHAPE
- PRACTICAL APPLICATIONS
- ROUTINE INCLUSION IN GENERAL-PURPOSE COMPUTER PROGRAMS

### LANGLEY PLANS

- EVALUATE GREEN'S FUNCTION METHOD FOR TRANSIENT TEMPERATURES
- DERIVATIVES WRT MATERIAL PROPERTIES
- IMPLEMENT FINITE DIFFERENCE STEP SIZE ALGORITHM
- SENSITIVITY DEMONSTRATION PROBLEMS

## SUMMARY

This paper was based on a recently conducted survey of methods for sensitivity analysis of structural response. The survey was not limited to research in the structural area alone and revealed that a broad range of disciplines are using sensitivity analysis and contributing to the methodology. In almost every instance, methods from the nonstructural disciplines are directly applicable to the structures area. An example application from chemical kinetics was described in which sensitivity analysis was used to assess the impact of parameter uncertainties in a mathematical model used in air pollution studies.

The bulk of the paper has focused on a selected set of innovative methods applicable to sensitivity analysis of structural systems. The analysis techniques include a finite difference step size selection algorithm, a method for derivatives of iterative solutions, a Green's function technique for derivatives of transient response, concurrent calculation of temperatures and their derivatives, derivatives with respect to shape, and derivatives of optimum designs with respect to problem parameters. Two applications were described wherein derivatives were used to guide structural design changes to improve an engineering design without recourse to formal mathematical optimization. Plans at Langley for contributing to identified critical needs were cited. Among the needs were implementation of methods for derivatives of transient response, derivatives with respect to shape and material properties, solution and documentation of sensitivity analysis demonstration problems and routine inclusion of sensitivity analysis as a feature in general purpose structural analysis computer programs. Langley near-term plans in the sensitivity area include evaluating the Green's function method for derivatives of transient thermal response, developing methods for derivatives with respect to material properties, implementation of the finite-difference step size algorithm, and solution of sensitivity demonstration problems.

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16. Abstract  This paper reviews some innovative techniques applicable to sensitivity analysis of discretized structural systems. The techniques include a finite difference step size selection algorithm, a method for derivatives of iterative solutions, a Green's function technique for derivatives of transient response, simultaneous calculation of temperatures and their derivatives, derivatives with respect to shape, and derivatives of optimum designs with respect to problem parameters. Computerized implementations of sensitivity analysis and applications of sensitivity derivatives are also discussed. Finally, some of the critical needs in the structural sensitivity area are indicated along with Langley plans for dealing with some of those needs.					
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